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A MONETARY EQUILIBRIUM MODEL WITH TRANSACTIONS COSTS

Julio J. Rotemberg*

August 1982

Working Paper #1347-82

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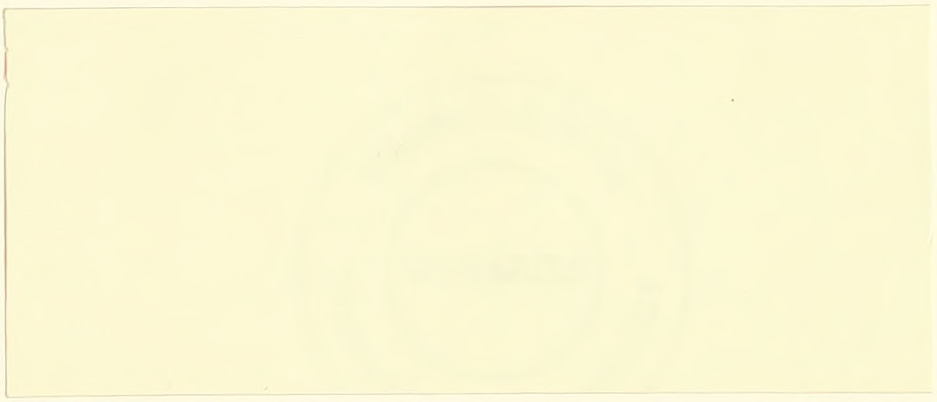

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ABSTRACT

This paper presents the competitive equilibrium of an economy in which people hold money for transactions purposes. It studies both the steady states which result from different rates of monetary expansion and the effects of such non steady state events as an open market operation. Even though the model features no uncertainty and perfect foresight, open market operations affect aggregate output. In particular, a simultaneous increase in money and governmental holdings of capital temporarily raises aggregate capital and output while it lowers the real rate of interest on capital.

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I. INTRODUCTION

The objective of this paper is to study the competitive equilibrium of an economy in which people hold money for transactions purposes. As in the models of Baumol (1952), Tobin (1956), Stockman (1981), Townsend (1982) and Jovanovic (1982), but in contrast to those of Grandmont and Younes (1973), Lucas (1980) and Helpman (1982), households are also allowed to hold interest bearing capital in addition to barren money. The main advantage of the present model is that it is able to shed light on the effects of such nonsteady state events as open market operations.

Households pick the path of consumption optimally from their point of view. Because it is costly to carry out financial transactions, people visit their financial intermediary only occasionally. However, I do not let households pick optimally the length of the period during which they do not visit their bank. For tractability, the assumption is made that households have a constant interval during which they carry out no financial transactions.

A crucial feature of this paper is that, as in all free market economies, different households visit their banks at different times. This leads to conclusions which are strikingly "Keynesian". Government interventions, and in particular open market operations, have the ability to affect aggregate output and the real rate of interest. This is true even though the model features no uncertainty, full information, perfect foresight and perfectly competitive markets for goods and money. Moreover, the effects of monetary policies closely resemble those found in standard textbooks. In particular, a one period monetary expansion leads to a higher level of output which persists for some time. It also concurrently leads to low real interest rates.

The paper proceeds as follows: Section II presents the model. It shows the maximization problems of the households and the firms, as well as the

institutional environment. Section III presents the perfect foresight equilibrium of the economy. It is a difference equation which exhibits saddle path stability near that steady state which has positive consumption. Section IV studies steady state inflation and welfare. It establishes that the level of output is independent of the rate of inflation but that inflation affects welfare negatively by distorting the intertemporal consumption decisions. Section V is the heart of the paper since it discusses monetary policy outside the steady state. Finally, Section VI presents some conclusions.

II. THE MODEL

There is only one good which can both be consumed and invested. Total output produced by firms in period t (Q_t) depends, via a constant returns to scale production function, on the amount of labor hired at t and on K_{t-1}, the amount of the good which was produced but not consumed at t-1. Since an amount of labor $\bar{L}$ is assumed to be supplied inelastically:

$$Q_t = \bar{L} f\left(\frac{K_{t-1}}{\bar{L}}\right) \tag{1}$$

where f is an increasing and concave function. Workers are assumed to be paid their marginal product. Therefore, the total return, denominated in period t goods, from foregoing the consumption of an additional good at t-1, is given by $1+r_{t-1}$, where:[1]

$$1 + r_{t-1} = f'\left(\frac{K_{t-1}}{\bar{L}}\right) \tag{2}$$

There are 2n households. At time t, the households are assumed to maximize the utility function given by:

$$V_t = \sum_{\tau=t}^{\infty} \rho^{\tau-t} \ln C_\tau^i \tag{3}$$

where C^i_τ is the consumption of household i at time τ, and ρ is a discount factor. The households have access to two assets, money and claims on capital. Money is the only medium of exchange. Moreover, visits to the financial intermediary for the purpose of converting claims on capital into money are costly. Therefore, as in the inventory theoretical models of Baumol (1952) and Tobin (1956), households engage in these visits only sporadically. In this paper it will be assumed at the outset that households exchange capital for money every two periods. The assumption that households do not change the timing of their financial transactions in response to events, is made mainly for tractability. Except in stationary environments, it is very difficult to solve for the optimal timing of these visits, particularly when households pick their consumption path optimally.

Without loss of generality, suppose that household i engages in financial transactions in the "even" periods, t, t+2, t+4,... . At these dates it withdraws an amount M^i_τ of money balances which must be sufficient to pay for its consumption at τ and $\tau+1$:

$$M^i_\tau = P_\tau C^i_\tau + P_{\tau+1} C^i_{\tau+1} \tag{4}$$

where P_τ is the nominal price of the consumption and investment good at τ. The evolution of K^i_τ, the claims on capital of household i at τ is given by:

$$K^i_{t+2k} + \frac{M^i_{t+2k}}{P_{t+2k}} + B = (1 + r_{t+2k-2})(1 + r_{t+2k-1})K^i_{t+2k-2}$$

$$+ (1+r_{t+2k-1})Y^i_{t+2k-1} + Y^i_{t+2k} \qquad k = 0,1,2,... \tag{5}$$

Here, B is the real cost of visiting the financial intermediary, while Y^i_τ is the noninvestment income of the household at τ. This noninvestment income

includes labor income as well as taxes and transfers from the government. Equation (5) says that investments minus brokerage fees at t+2k have to be equal to total resources at t+2k. These resources include the capitalized values of K^i_{t+2k-2} and of noncapital income at t+2k-1, as well as current noncapital income. Note that (5) explicitly assumes that noncapital income is directly invested in claims on capital. This assumption considerably simplifies the analysis.

The optimal path of consumption is found in two steps. First, I derive consumption at τ and $\tau+1$ as a function of M^i_τ. Then I derive the optimal values of the sequence of monetary withdrawals.

The first step requires the maximization of:

$$\ln C^i_\tau + \rho \ln C^i_{\tau+1} \tag{6}$$

subject to (4). This yields:[2]

$$C^i_{\tau+1} = \frac{\rho P_\tau}{P_{\tau+1}} C^i_\tau = \frac{\rho}{1+\rho} \frac{M^i_\tau}{P_{\tau+1}} \tag{7}$$

Using (7), the expression in (6) is given by:

$$\ln C^i_\tau + \rho \ln C^i_{\tau+1} = (1+\rho)\ln\left(\frac{M^i_\tau}{P_\tau}\right) + \rho\ln\rho - (1+\rho)\ln(1+\rho) - \rho\ln\left(\frac{P_{t+1}}{P_t}\right) \tag{8}$$

Equation (8) asserts that the appropriately weighted sum of the instantaneous utilities at τ and $\tau+1$ increases with M^i_τ / P_τ but is negatively affected by inflation between τ and $\tau+1$.

Substituting (8) into (3) and using (5), one obtains:

$$V_t = \sum_{k=0}^{\infty} \rho^{2k} \{\ln[(1+r_{t+2k-2})(1+r_{t+2k-1})K^i_{t+2k-2} + (1+r_{t+2k+1})Y_{t+2k-1}$$

$$+ Y_{t+2k} - B - K^i_{t+2k}] - \rho \ln \rho + (1+\rho)\ln(1+\rho) - \rho \ln\left(\frac{P_{t+2k+1}}{P_{t+2k}}\right)\} \quad (9)$$

This expression must now be maximized with respect to the sequence of claims on capital. This maximization yields:[3]

$$-\frac{1}{M^i_{t+2k}/P_{t+2k}} + \frac{\rho^2(1+r_{t+2k})(1+r_{t+2k+1})}{M^i_{t+2k+2}/P_{t+2k+2}} = 0 \quad (10)$$

Using (7), (10) becomes:

$$\frac{C^i_{t+2k+2}}{C^i_{t+2k}} = \rho^2(1+r_{t+2k})(1+r_{t+2k+1}) \quad (11)$$

Note that both (11) and (7) state that the marginal rate of substitution times a rate of return is equal to one. The important distinction between the two is that in (7) the rate of return is the rate of return on money, while in (11) it is the rate of return on capital. Stochastic versions of (11) have been statistically rejected using aggregate U.S. data by Mankiw (1981) and Hansen and Singleton (1982). Their rejections may be due in part to their neglect of the fact that in the presence of the transactions motive for holding money, the ratio of marginal utilities of consumption separated by different time intervals is related to rates of return of assets with different characteristics.

Financial intermediaries receive the household's income and invest it in claims on capital. They are also allowed to issue a certain quantity of money. The intermediaries are compensated for their services with the brokerage fees, B, of (5). Their function can best be understood by following their transactions in detail.

Between periods τ and $\tau+1$ the financial intermediaries have as their assets the household's claims on capital as well as M_τ units of money.

These can be thought of as deposits at the Federal Reserve Bank. Their liabilities are the household's claims on capital and the amount of money, $\tilde{M}_\tau$, that the households who came to the bank at τ withdrew but did not spend at τ. In period $\tau+1$, the financial intermediaries issue whatever amount of money the households who visit them in period $\tau+1$ require. The households then buy goods from the firms. The firms return the money they receive from the households to the intermediaries in partial payment of their compensation to workers and their debt to the households. The rest of their obligations to the banks is then paid in the form of claims on $\tau+1$ capital. Since the firms have constant returns to scale, their total obligations towards the households are given by:

$$L f\left(\frac{K_\tau}{L}\right) - f'\left(\frac{K_\tau}{L}\right) K^G_\tau = Y^L_\tau + K^P_\tau f'\left(\frac{K_\tau}{L}\right) \qquad (12)$$

where K^G_τ is the amount of capital owned by the government at τ, K^P_τ is the amount owned by the private sector which is equal to $K_\tau - K^G_\tau$ and Y^L_τ is labor income at τ. The sales of firms at $\tau+1$ are given by aggregate consumption at $\tau+1$, $C_{\tau+1}$. So the debt of firms held by households at the end of $\tau+1$ must be equal to:

$$L f\left(\frac{K_\tau}{L}\right) - C_{\tau+1} - K^G_{\tau+1} \qquad (13)$$

I also assume that half the households (n) visit the intermediaries in the even periods t,t+2,t+4,..., and the other half carry out their financial transactions in the odd periods. The fact that only a subset of the households visit financial intermediaries in any given day is one of the main features of reality which this paper seeks to reflect. It also is a feature of the steady states studied by Jovanovic (1982). It turns out that the assumption that households stagger their financial transactions is crucial to

ensure that open market operations have real effects.

The government in this model has no expenditures. However, it levies taxes, issues money, and holds capital. The evolution of the capital held by the government is given by:[4]

$$K^G_{\tau+1} = f'(K_\tau)K^G_\tau + \frac{M_{\tau+1} - M_\tau}{P_{\tau+1}} + T_{\tau+1} \qquad (14)$$

where $T_{\tau+1}$ are the real taxes levied at $\tau+1$. An increase in $M_{\tau+1}$ relative to M_τ will be called an open market purchase and therefore the domain of monetary policy. Instead, a simultaneous change of $T_{\tau+1}$ and $K^G_{\tau+1}$ will be considered a type of fiscal policy. The government also requires that between periods τ and $\tau+1$, the monetary liabilities of the financial intermediaries $\tilde{M}_\tau$ be equal to their monetary assets, M_τ.

III. EQUILIBRIUM

The equilibrium for this economy is a path for the price level and for the real rate of interest which when households maximize utility and firms maximize profits ensures that:

a) the sum of consumption and capital demanded at τ by the households, and capital demanded by the government at τ is equal to output at τ.

$$C_\tau + K^P_\tau + K^G_\tau = \bar{L} f\left(\frac{K_{\tau-1}}{\bar{L}}\right) \qquad (15)$$

b) The amount of money that households who visit the intermediary at τ want to hold between τ and $\tau+1$ must be equal to M_τ. Hence, the total expenditures at $\tau+1$ by households who visit the financial intermediary at τ must be equal to M_τ.

Let C^τ_τ and $C^{\tau-1}_\tau$ be the consumptions at τ of households who visit the financial intermediary at τ and $\tau-1$ respectively. Then condition (b) requires that:

$$nP_\tau C_\tau^{\tau-1} = M_{\tau-1} \tag{16}$$

Therefore, using (7):

$$n\rho P_\tau C_\tau^{\tau} = M_\tau \tag{17}$$

Therefore:

$$C_\tau^{\tau-1} = \frac{\rho M_{\tau-1}}{M_\tau} C_\tau^{\tau} \tag{18}$$

and aggregate consumption at τ, C_τ is given by:

$$C_\tau = n\left(1 + \frac{\rho M_{\tau-1}}{M_\tau}\right) C_\tau^{\tau} \tag{19}$$

Using (19), (11), and the equilibrium condition (15), one obtains the difference equation which governs the evolution of aggregate capital:

$$\bar{L}f\left(\frac{K_{\tau+2}}{\bar{L}}\right) - K_{\tau+3} = \rho^2 \frac{1+\rho\frac{M_{\tau+2}}{M_{\tau+3}}}{1+\rho\frac{M_\tau}{M_{\tau+1}}} f'(K_{\tau+1})f'(K_{\tau+2})\left[\bar{L}f\left(\frac{K_\tau}{\bar{L}}\right) - K_{\tau+1}\right]$$

$$\tau = t-1,\ t,\ t+1,\ldots \tag{20}$$

This equation completely characterizes the equilibria. Knowledge of the sequence of capitals provides the sequence of rates of return by (2), the aggregate consumptions by (15), the sequence of individual consumptions by (19) and the sequence of prices by (12). The equilibrium is thus a third order nonlinear difference equation with only one initial condition, namely K_{t-1}. There are therefore an infinite number of solutions. In particular, there are two arbitrary initial conditions, K_t and K_{t+1}. Alternatively one can choose C_t^t and C_{t+1}^{t+1} at will. The standard neoclassical optimum growth problem is simply the problem of this paper but with the requirement that people can spend their claims on capital every period. This standard problem has only one free initial condition, namely consumption in the first period.

Here, since I am dealing with two types of consumers situated rather differently two arbitrary initial conditions are required. The question remains whether there exists a unique equilibrium which converges to a steady state with positive consumption. As long as $M_\tau/M_{\tau+1}$ converges to a constant, the steady state values of capital (K*) have the following property:

$$[\bar{L}f(\frac{K^*}{\bar{L}}) - K^*]\{1 - \rho^2[f'(\frac{K^*}{\bar{L}})]^2\} = 0$$

The steady state values of K do not depend on the rate of monetary growth. There are two types of steady states. Those with zero consumption are such that output $\bar{L}f(K^*/\bar{L})$ is equal to investment, K*. The only steady state with positive consumption has the property that $\rho f'(K) = 1$; the product of the discount rate times the marginal product of capital is unity. Unfortunately, I cannot establish the existence or uniqueness of paths which converge to $\bar{K}$. I can only present a local result. Namely, that the linearized version around $\bar{K}$ of (20) is such that a unique path which converges to $\bar{K}$ exists. The linearized version of (20) around $\bar{K}$ is:

$$(K_{t+3} - \bar{K}) - \{f'(\bar{K}) - \rho[f(\frac{\bar{K}}{\bar{L}}) - \frac{\bar{K}}{\bar{L}}]f''(\frac{\bar{K}}{\bar{L}})\}(K_{t+2} - \bar{K}) - \{1 - \rho[f(\frac{\bar{K}}{\bar{L}}) - \frac{\bar{K}}{\bar{L}}]f''(\frac{\bar{K}}{\bar{L}})\}$$

$$(K_{t+1} - \bar{K}) + f'(\frac{\bar{K}}{\bar{L}})(K_t - \bar{K}) = 0 \qquad (21)$$

The homogeneous part of equation (21) can be written as the following polynominal in the lag operator L:

$$(1 - \lambda_1 L)(1 - \lambda_2 L)(1 - \lambda_3 L)K_\tau = 0 \qquad (22)$$

where the roots λ_1, λ_2, and λ_3 have the following properties:

$$\lambda_1\lambda_2\lambda_3 = -f'(\frac{K}{\bar{L}})$$

$$\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_1\lambda_3 = -1 + \rho[f(\frac{\bar{K}}{\bar{L}}) - \frac{\bar{K}}{\bar{L}}]f''(\frac{\bar{K}}{\bar{L}}) \qquad (23)$$

$$\lambda_1 + \lambda_2 + \lambda_3 = f'(\frac{\bar{K}}{\bar{L}}) - \rho[f(\frac{\bar{K}}{\bar{L}}) - \frac{\bar{K}}{\bar{L}}]f''(\frac{\bar{K}}{\bar{L}})$$

Inspection of these equations reveals that one of the roots, say λ_1, is equal to minus one, while the other two roots are such that $(\lambda_2-1)(\lambda_3-1)$ is negative, $\lambda_2\lambda_3$ is positive while $(\lambda_2+\lambda_3)$ is positive. Therefore, λ_2 and λ_3 are both positive and lie on opposite sides of the unit circle. There is only one stable root, say λ_3. λ_2 is such that if the initial conditions are not chosen correctly, K either explodes or implodes. Finally, λ_1 such that unless the initial conditions are appropriately chosen, capital is subject to oscillations in the steady state. The arguments of Blanchard and Kahn (1980) establish that for a unique nonexplosive equilibrium to exist, the number of roots at or inside the unit circle must be equal to the number of predetermined variables. Here there are two roots at or inside the unit circle and only one predetermined variable. So there exists an infinity of nonexplosive paths. However, there is only one path which does not oscillate in the steady state.

IV. INFLATION, STEADY STATES AND WELFARE

In Section III I established that there is no Tobin effect in this model. Independently of the value of M_{t+1}/M_t, the unique steady state value of the capital stock which involves positive consumption is $\bar{K}$. This does not mean that inflation has no real effects. In particular, the steady state rate of growth of the money stock affects the path of individual consumption, the level of welfare, and the income velocity of money.

Before studying the effects of inflation, however, it must be

ascertained that an inflationary path is consistent with the government's budget constraint (14). I will assume that the government lets the money stock grow at the rate m so that $M_\tau/M_{\tau-1} = 1 + m$. With the new money, the government buys capital which it redistributes in lump sum fashion. These lump sum redistributions affect none of the conditions used to derive (20). So the amount of capital held by the government can be arbitrarily set to some constant.

I now compute the rate of inflation which corresponds to a given rate of growth of money. Equation (17) establishes that the steady state rate of inflation, π, is given by:

$$\frac{P_{\tau+1}}{P_\tau} = 1 + \pi = \rho \frac{C_\tau^\tau}{C_\tau^{\tau-1}} \tag{24}$$

where, in the steady state, neither C_τ^τ nor $C_\tau^{\tau-1}$ depend on τ. Therefore, using (18):

$$1 + \pi = \frac{M_\tau}{M_{\tau-1}} = 1 + m \tag{25}$$

The rate of inflation is equal to the rate of monetary expansion. Note that the model is quite consistent with the rate of return dominance of capital over money in the steady state. The rate of return of the former is $[(1/\rho) - 1]$, while that of the latter is $-m$.

By (24), the ratio of consumption on the date of financial transactions to consumption in the following period is: $(1+m)/\rho$; it rises as inflation rises. Therefore, inflation distorts the intertemporal consumption decisions. It leads people to consume more right after they withdraw money and less in later periods.[5] The rate of deflation which is such that intertemporal consumption decisions are optimal from the point of view of society is $[(1/\rho) - 1]$. As Friedman (1969) proposed, the rate of deflation must be

equal to the discount rate. This result was also obtained by Jovanovic (1982) in a model in which money is held for transactions purposes but in which people, while picking the timing of their visits to the financial intermediaries optimally, do not pick their consumption path optimally. Here, it can be shown as follows: consider a social planner who wants to maximize:

$$\sum_{\tau=1}^{\infty} \rho^{\tau-t}[\alpha \ln C_{\tau}^{\tau} + (1 - \alpha)\ln C_{\tau}^{\tau-1}] \tag{26}$$

subject to:

$$K_{\tau} = \bar{L} f\left(\frac{K_{\tau-1}}{\bar{L}}\right) - n(C_{\tau}^{\tau} + C_{\tau}^{\tau-1}) - nB \tag{27}$$

where α is a weight between zero and one. This social planner maximizes a convex combination of the utilities of both types of consumers subject to society's budget constraint. The plan which maximizes (26) satisfies:

$$C_{\tau}^{\tau} = \frac{\alpha}{1-\alpha} C_{\tau}^{\tau-1}$$

and:

$$C_{\tau+1}^{\tau} = \rho f'\left(\frac{K_{\tau-1}}{\bar{L}}\right) C_{\tau}^{\tau} \tag{28}$$

In the steady state in which $\rho f'(K_{\tau}/\bar{L})$ is equal to one, consumption is constant. Indeed, this is precisely what occurs in the decentralized economy with money as long as, in (7) $\rho P_{t+1}/P_t$ is equal to one. This in turn requires that m be equal to $[1 - (1/\rho)]$ as claimed.

However, there is a problem in sustaining the equilibrium with $K = \bar{K}$ when m is equal to $[1 - (1/\rho)]$. This problem arises because at this equilibrium money and capital have the same rate of return. Therefore households would prefer to withdraw money at the beginning of their lives in the amount equal to the present discounted value of their income. Then they would avoid all

visits to their bank and associated transactions costs. This would result in no capital being available in this model with 100% reserves. It may well be the case that, with a smaller reserve requirement, the equilibrium would be sustainable. In any event, note that, as long as m is just slightly bigger than $-[(1/\rho) - 1]$, the equilibrium of this model is essentially equal to the Pareto optimum and does induce people to hold $\bar{K}$ units of capital.

Inflation induces people to consume less in the period in which they do not go to the bank. Therefore, $M_\tau/P_{\tau+1}$, which is equal to this consumption, falls. However, surprisingly, in this model, M_τ/P_τ actually rises with inflation. This result is undoubtedly due in part to the fact that people do not go to the bank more often when inflation rises. It emerges because even though people want to reduce $M_\tau/P_{\tau+1}$, they must increase M_τ/P_τ to ensure that inflation does not reduce $M_\tau/P_{\tau+1}$ too much. The result can be established by noting that equation (18) says that M_τ/P_τ is proportional to consumption in the period in which people visit the financial intermediary. By equation (19), this consumption does indeed rise with inflation.

V. THE NONNEUTRALITY OF MONETARY POLICY

The main purpose of this paper is to study conditions outside the steady state. First, it will be established that a wide variety of monetary policies (or open market operations) affect aggregate output. This is the consensus view of textbooks, such as Branson (1979). However, this view has recently been challenged by a variety of authors (e.g., Wallace (1981), Chamley and Polemarchakis (1982)). These authors have shown that in models in which money is held only for its rate of return characteristics, open market operations are neutral. Admittedly the premise of these models -- that money is not rate of return dominated by other assets -- appears to be a bad description of

free-market economies as we know them. The proof that open market operations can affect output in the economy of this paper is straightforward. Consider a base path for money and taxes: $\{\tilde{M}_\tau\}$ and $\{\tilde{T}_\tau\}$, then an equilibrium sequence of capital $\{\tilde{K}_\tau\}$ must satisfy (21). Consider any one of these equilibria.

Now consider a slightly different financial policy for the government. At t, unexpectedly, the government purchases some extra capital by issuing ε units of money. Then, at T, the government engages in the reverse transaction: it sells $\varepsilon \prod_{\tau=t}^{T-1} f'(\frac{K_\tau}{L}) / P_t$ units of capital. Therefore, the path of taxes remains unchanged, but between t and T the path of money is replaced by $\{M_\tau + \varepsilon\}$. After T, the path of money is given by: $\{\tilde{M}_\tau + \varepsilon[1 - P_T \prod_{i=t}^{T} f'(\frac{K_i}{L})] / P_t\}$. Then, it is clear from (21) that if, in the new equilibrium, capital remains unchanged from $\{\tilde{K}_\tau\}$ at t and t+1, it will have to be different from $\tilde{K}_{t+2}$ at t+2; monetary policy affects output.

The intuition behind this result is as follows. Suppose the open market operation has no effect on prices. Then, the paths of consumption will be unaffected. However, the people who visit the bank at t will not want to demand the increased amount of money balances. If, instead, any rate of inflation after t is affected by the open market operation, then by (7) someone will change their consumption path. Finally, suppose that only the price at t is affected by the government's change of financial policy. Then the consumption of those who visited the bank at t-1, which is given by M_{t-1}/P_t will be affected. So the nonneutrality of open market operations hinges crucially on the fact that not everyone visits the bank on the day of the operation. This is, in fact, a striking feature of the U.S. institutional setup.

I am not just interested in establishing that monetary policy is non-neutral. Instead, I want to characterize their effect. I am unable to do so

for general policies and general production functions. Instead, the rest of this section is devoted to simulating various monetary policies under additional assumptions.

First, I will assume that the production function is Cobb-Douglas, and I will normalize the aggregate labor endowment to be equal to one. Therefore:

$$\omega_t = K_{t-1}^{\alpha} \tag{29}$$

Moreover, I will assume that $\alpha = 0.25$, and that $\rho = 0.99$. This high rate of discount is appropriate since people visit financial intermediaries often.

I pick as the relevant equilibrium the path which converges to $\bar{K}$ which, in this case, is equal to 0.1553938. I consider in particular an expansion of the money stock at time one from 100 to 119.9. The capital at time zero is assumed to be $\bar{K}$. Then, at time 11, the government is assumed to sell back all the capital it bought in time period 2. In equilibrium, this involves approximately a 20% fall in the money stock. Figure 1 shows the path of capital for this experiment, while Figure 2 shows the path of real interest rates. As these figures indicate, the effects are not negligible. Capital increases almost 4% after the original open market purchase. This naturally raises output. Moreover, this increase is accompanied by negative real rates of interest. Note also that the monetary contraction of period 11 is accompanied by a fall in capital that period, even though that contraction is predicted by agents as of period 1. The intuition behind these results is the following. When the quantity of money is increased at τ, the price level rises. This decreases $M_{\tau-1}/P_{\tau}$ and therefore reduced $C_{\tau}^{\tau-1}$. This fall in the consumption by those who do not visit their bank at τ, raises capital and hence output in the following period. This explanation suggests that monetary policy may derive much of its power in this model from the assumption that

those people who visited the bank at τ-1 do not change their pattern of bank visits in response to inflation at τ. How much people who had not scheduled a visit to their intermediary at τ would reduce their consumption in response to inflation at τ if they were free to pick the timing of these visits optimally, is an open question which deserves further research.

Unfortunately, the figures show that capital converges very slowly to the steady state and that the negative root has important effects on the dynamics of the economy. For purposes of comparison, I also simulate the effects from a once and for all expansion of money in period one to 119.9 from 100. This monetary expansion is then followed by the lump sum distribution of tne purchased capital. Figure 3 shows the resulting path for capital. What is striking about this path is that the expansion of capital in the early periods is almost identical to the expansion in Figure 2. Monetary policy understood here as financial policy has about the same power as an expansion in money which is unaccompanied by new government assets.

Before concluding this section, it is worth noting that if the rate of monetary expansion is changed unexpectedly once and for all at t, the path of capital is unaffected. Suppose that before t, $M_{\tau+1}/M_{\tau}$ was equal to $(1+\bar{m})$. Then, before the change in rates of monetary growth, (21) asserts that the evolution of the capital stock was given by:

$$K_{\tau+3} = \bar{C}f\left(\frac{K_{\tau+2}}{\bar{C}}\right) - \rho^2 f'\left(\frac{K_{\tau+1}}{\bar{C}}\right) f'\left(\frac{K_{\tau+2}}{\bar{C}}\right)\left[\bar{C}f\left(\frac{K_{\tau}}{\bar{C}}\right) - K_{\tau+1}\right] \qquad (29)$$

Now, suppose that at t, it is announced that from now on $M_{\tau+1}/M_{\tau}$ will be given by $(1+m^*)$. Then the evolution of capital from t+3 on is still given by (29), and the original values of K_{t+1} and K_{t+2} are still equilibrium values of capital.

VI. CONCLUSIONS

The model of this paper is a modest step towards the construction of tractable and realistic general equilibrium models capable of shedding light on the effects of nonsteady-state monetary changes. Its major advantage is that people's motive for holding money is explicitly that money is used for transactions. In particular, in those periods in which households do not visit their banks, they are faced with an extreme version of the "Clower Constraint"; they must pay for their purchases with money carried over from the previous period. This ensures that monetary policies which expand money and prices, reduce the real consumption of those households which do not visit their financial intermediary on the day of the monetary expansion. This fall in consumption raises capital and output in future periods.

A number of issues are raised by this paper. First, an important question is to what extent the power of monetary policy would be diluted if people timed their visits to intermediaries optimally. Associated with this question is the question of whether people in fact do significantly alter the interval during which they refrain from visiting their bank as events change.

The framework of this paper can also hopefully be used to study the effects of various institutional setups on macroeconomic activity. In particular, it should be capable of shedding some light on the difference between commodity standards, fractional reserve standards and the 100% reserves standard of this paper.

Figure 1

Movements of Capital after an Open Market Purchase

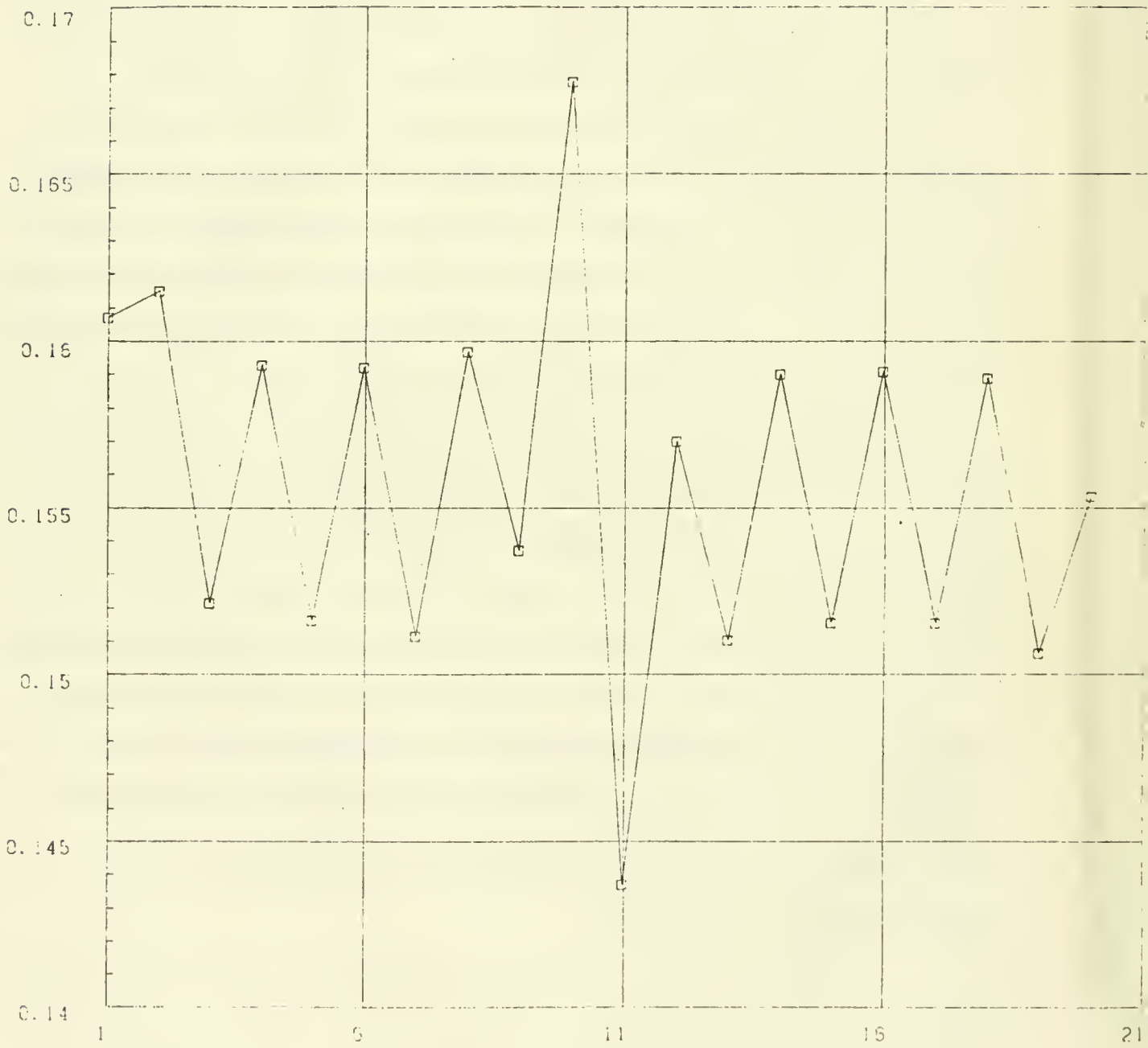


Figure 2

Movement of the Real Interest Rate
After an Open Market Purchase

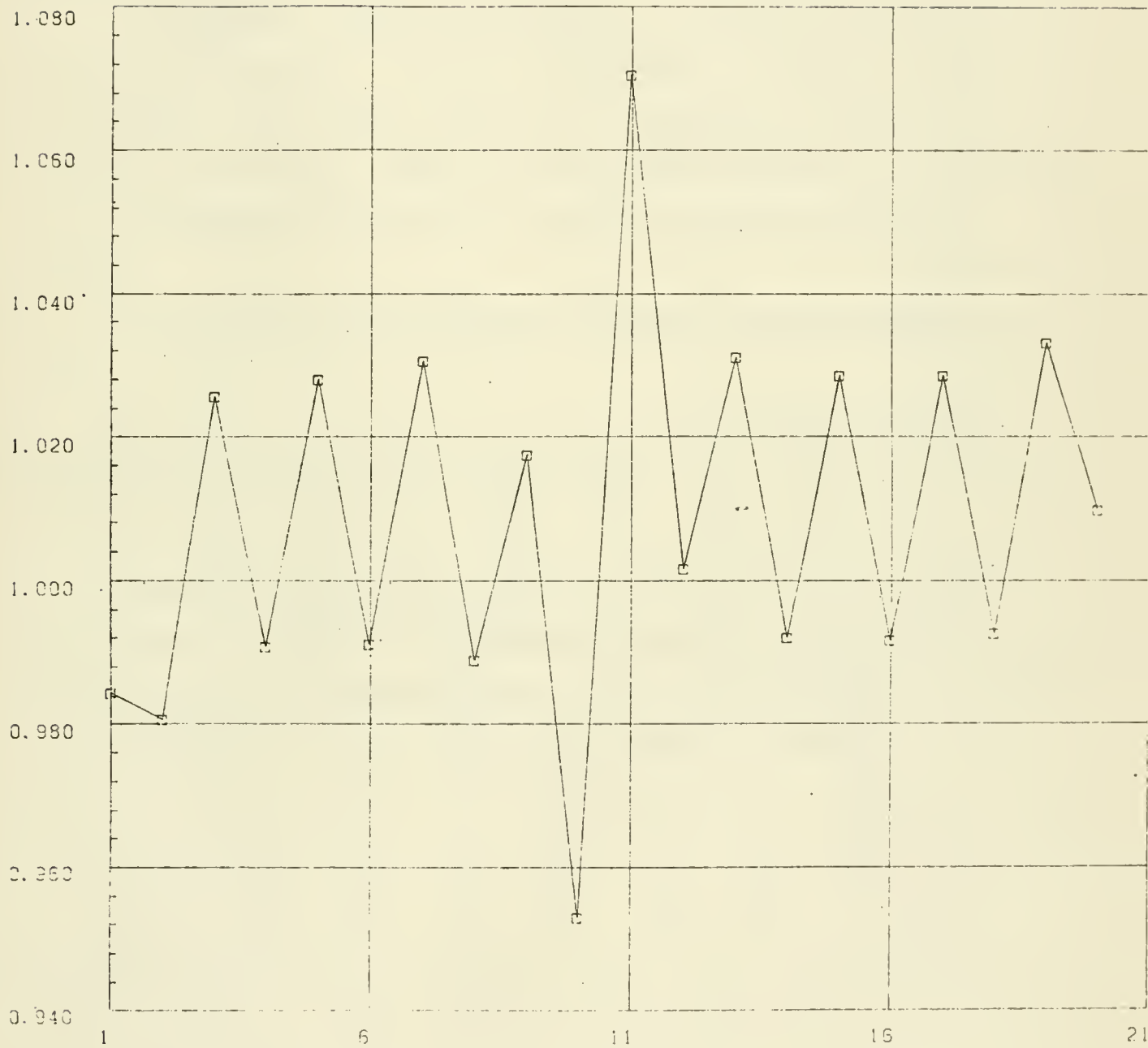
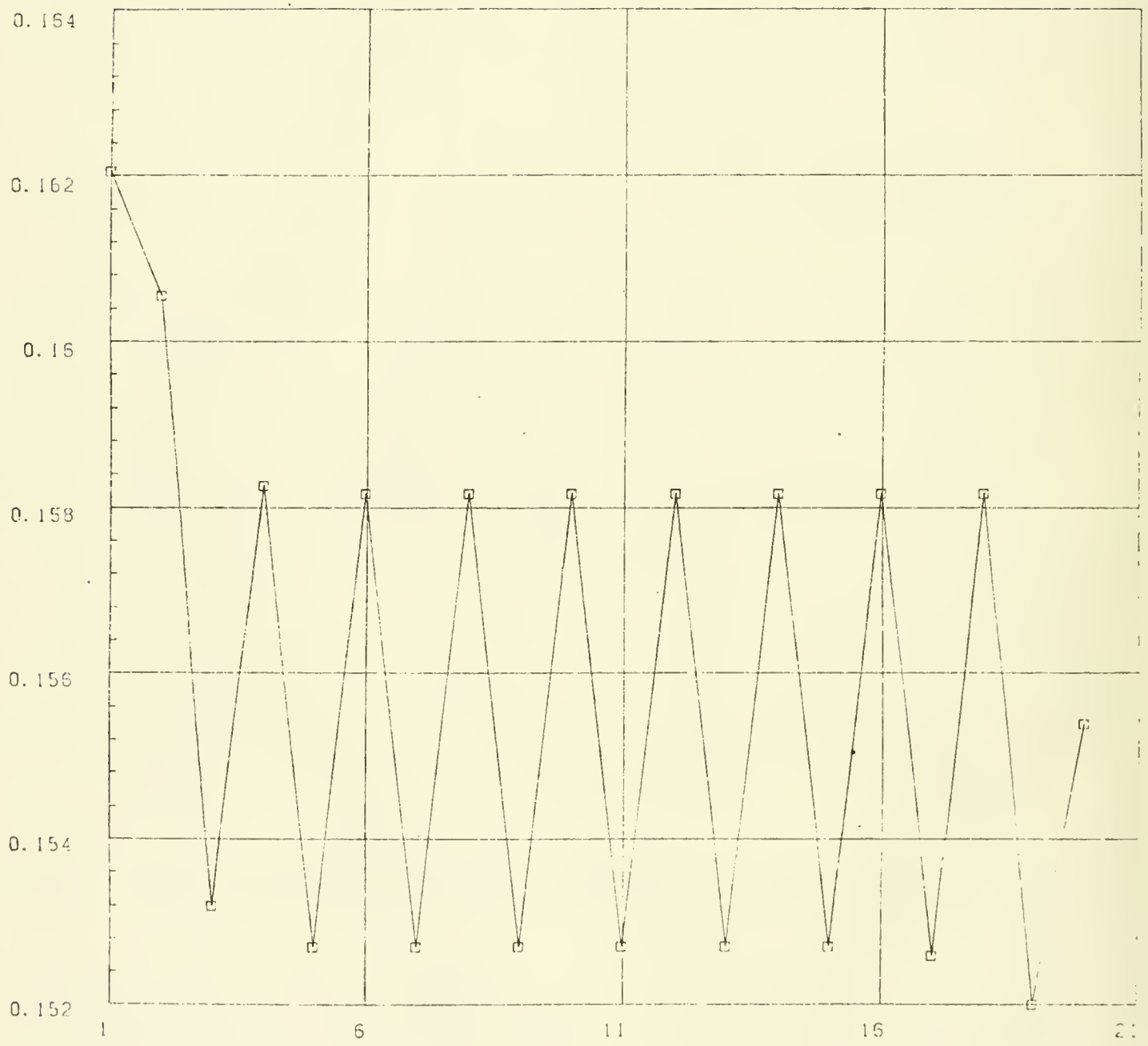


Figure 3

Movements of Capital After an Expansion in Money



FOOTNOTES

[1]Primes denote first derivatives, while double primes denote second derivatives.

[2]The analysis assumes that the holdings of money by household i from τ to $\tau+1$ are nonnegative. Here this is guaranteed by the fact that negative holdings of money would induce negative consumption and hence utility equal to minus infinity. However, it labor income were paid in the form of money, the constraint that monetary holdings be nonnegative might become binding.

[3]This condition requires that capital be held in positive amounts at $t+2k$.

[4]For simplicity, ignore the transaction costs incurred by the government when it engages in an open market operation.

[5]This has been noted also by Jovanovic (1982). This effect is likely to be even more pronounced on consumer expenditure when there are durable goods, and consumer expenditure can be different from consumption. In this paper, consumption and consumer expenditure coincide by assumption.

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